Definition and Formal Metatheory of ABS

Abstract

We define ABS and describe its metatheory.

Contents

1 Introduction

Introduction goes here.

2 Syntax

In this section, we define the abstract syntax of ABS.

T	::=		ground type
	|	Bool	
	|	Int	

F	::=		function definition
	|	$\textbf{def}\ T\, fn(T_1\, x_1,\, \ldots,\, T_n\, x_n) = e;$	

t	::=		ground term
	|	b	boolean
	|	z	integer

e	::=		expression
	|	t	term
	|	x	variable
	|	$fn(e_1,\, \ldots,\, e_n)$	function call
	|	$e[x_1 \mapsto y_1,\, \ldots,\, x_n \mapsto y_n]$	M

3 Semantics

In this section, we define the semantics of ABS.

3.1 Meta syntax

sig	::=	
	|	$T_1,\, \ldots,\, T_n \to T$

$ctxv$	::=	
	|	T
	|	sig

3.2 Typing

$\boxed{\Gamma \vdash e : T}$ well-typed expression

$$\frac{}{\Gamma \vdash b : \mathsf{Bool}} \quad \text{TYP\_BOOL}$$

$$\frac{}{\Gamma \vdash z : \mathsf{Int}} \quad \text{TYP\_INT}$$

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \quad \text{TYP\_VAR}$$

$$\frac{\begin{array}{c}\Gamma \vdash e_1 : T_1 \quad \ldots \quad \Gamma \vdash e_n : T_n \\ \Gamma(fn) = T_1, \ldots, T_n \to T\end{array}}{\Gamma \vdash fn(e_1, \ldots, e_n) : T} \quad \text{TYP\_FUNC\_EXPR}$$

$\boxed{\Gamma \vdash F}$ well-typed function declaration

$$\frac{\begin{array}{l}\Gamma(fn) = T_1, \ldots, T_n \to T \\ \Gamma[x_1 \mapsto T_1, \ldots, x_n \mapsto T_n] \vdash e : T \\ \textbf{distinct}\,(x_1, \ldots, x_n)\end{array}}{\Gamma \vdash \textbf{def}\ T\ fn(T_1\ x_1, \ldots, T_n\ x_n) = e;} \quad \text{TYP\_FUNC\_DECL}$$

3.3 Reduction

$\boxed{F_1 \ldots F_n, \sigma \vdash e \leadsto \sigma' \vdash e'}$ expression evaluation

$$\frac{\sigma(x) = t}{F_1 \ldots F_n, \sigma \vdash x \leadsto \sigma \vdash t} \quad \text{RED\_VAR}$$

$$\frac{F_1 \ldots F_n, \sigma \vdash e \leadsto \sigma' \vdash e'}{F_1 \ldots F_n, \sigma \vdash fn(e_1, \ldots, e_i, e, e'_1, \ldots, e'_j) \leadsto \sigma' \vdash fn(e_1, \ldots, e_i, e', e'_1, \ldots, e'_j)} \quad \text{RED\_FUN\_EXP}$$

$$\frac{\begin{array}{l}\textbf{well\_formed}\,(y_1, \ldots, y_n, e, \sigma) \\ \textbf{disjoint}\,((y_1, \ldots, y_n), (x_1, \ldots, x_n))\end{array}}{F_1 \ldots F_i\ \textbf{def}\ T\ fn(T_1\ x_1, \ldots, T_n\ x_n) = e;\ F'_1 \ldots F'_j, \sigma \vdash fn(t_1, \ldots, t_n) \leadsto \sigma[y_1 \mapsto t_1, \ldots, y_n \mapsto t_n] \vdash e[x_1 \mapsto y_1, \ldots, x_n \mapsto y_n]}$$

4 Metatheory

In this section, we define the metatheory of ABS.

Definition. A context Γ' subsumes a context Γ, written $\Gamma \subseteq \Gamma'$, if (1) whenever $\Gamma(x) = T$ then $\Gamma'(x) = T$ and (2) whenever $\Gamma(fn) = sig$ then $\Gamma'(fn) = sig$.

Definition. A context Γ is consistent with a substitutition σ, written $\Gamma \vdash \sigma$, if whenever $\sigma(x) = t$ and $\Gamma(x) = T$, then $\Gamma \vdash t : T$.

Theorem 1 (Type preservation)**.** Assume $\Gamma \vdash F_1 \quad \ldots \quad \Gamma \vdash F_n$ and $\Gamma \vdash \sigma$. If $\Gamma \vdash e : T$ and $F_1 \ldots F_n, \sigma \vdash e \leadsto \sigma' \vdash e'$, then there is a Γ' such that $\Gamma \subseteq \Gamma'$, $\Gamma' \vdash \sigma'$, and $\Gamma' \vdash e' : T$.