

$n$  number  
 $x$  variable  
 $e$

$::=$   
 $|$   $x$  variable  
 $|$   $n$  number  
 $|$   $e + e'$  plus  
 $|$   $e * e'$  times  
 $|$  **let**  $x := e$  **in**  $e'$  bind  $x$  in  $e'$  let  
 $|$   $e[e'/x]$  M substitution  
 $|$   $(e)$  S

*terminals*  $::=$   
 $|$   $\rightarrow$   
 $|$   $\Downarrow$

*formula*  $::=$   
 $|$  *judgement*  
 $|$   $n + n' = n''$  M  
 $|$   $n * n' = n''$  M

*operational\_semantics*  $::=$   
 $|$   $e \rightarrow e'$  reduction step  
 $|$   $e \Downarrow n$  evaluates to

*judgement*  $::=$   
 $|$  *operational\_semantics*

*user\_syntax*  $::=$   
 $|$   $n$   
 $|$   $x$   
 $|$   $e$   
 $|$  *terminals*  
 $|$  *formula*

$\boxed{e \rightarrow e'}$  reduction step

$$\frac{n_1 + n_2 = n}{n_1 + n_2 \rightarrow n} \text{ OS\_RED\_PLUS}$$

$$\frac{e_1 \rightarrow e'_1}{e_1 + e_2 \rightarrow e'_1 + e_2} \text{ OS\_RED\_PLUS\_L}$$

$$\frac{e \rightarrow e'}{n + e \rightarrow n + e'} \text{ OS\_RED\_PLUS\_R}$$

$$\frac{n_1 * n_2 = n}{n_1 * n_2 \rightarrow n} \text{ OS\_RED\_TIMES}$$

$$\frac{e_1 \rightarrow e'_1}{e_1 * e_2 \rightarrow e'_1 * e_2} \text{ OS\_RED\_TIMES\_L}$$

$$\frac{e \rightarrow e'}{n * e \rightarrow n * e'} \text{ OS\_RED\_TIMES\_R}$$

$$\frac{e_1 \rightarrow e'_1}{\mathbf{let } x := e_1 \mathbf{ in } e_2 \rightarrow \mathbf{let } x := e'_1 \mathbf{ in } e_2} \quad \text{OS\_RED\_LET}$$

$$\frac{}{\mathbf{let } x := n \mathbf{ in } e_2 \rightarrow e_2[n/x]} \quad \text{OS\_RED\_BIND}$$

$e \Downarrow n$  evaluates to

$$\frac{}{n \Downarrow n} \quad \text{OS\_EVAL\_NUM}$$

$$\frac{\begin{array}{l} e_1 \Downarrow n_1 \\ e_2 \Downarrow n_2 \\ n_1 + n_2 = n \end{array}}{e_1 + e_2 \Downarrow n} \quad \text{OS\_EVAL\_PLUS}$$

$$\frac{\begin{array}{l} e_1 \Downarrow n_1 \\ e_2 \Downarrow n_2 \\ n_1 * n_2 = n \end{array}}{e_1 * e_2 \Downarrow n} \quad \text{OS\_EVAL\_TIMES}$$

$$\frac{\begin{array}{l} e_1 \Downarrow n_1 \\ e_2[n_1/x] \Downarrow n_2 \end{array}}{\mathbf{let } x := e_1 \mathbf{ in } e_2 \Downarrow n_2} \quad \text{OS\_EVAL\_LET}$$

Definition rules: 12 good 0 bad

Definition rule clauses: 27 good 0 bad