

var, x, y	term variable		
t	::=		term
		x	variable
		$\lambda x.t$	bind x in t lambda
		$t t'$	app
		(t)	S
		$[t/x]t'$	M
v	::=		value
		$\lambda x.t$	lambda
typ, T	::=		types
		\circ	base type
		$T_1 \rightarrow T_2$	function types
ctx, Γ	::=		typing context
		\bullet	empty context
		$\Gamma, x : T$	assumption
$terminals$::=		
		λ	
		\longrightarrow	
		\rightarrow	
		\in	
		\neq	
		\equiv_α	
		\equiv_β	
		FV	
		\notin	
		dom	
		\vdash	
$formula$::=		
		$judgement$	
		$x \neq x'$	M
		$x \notin \text{FV}(t)$	M
		$x : T \in \Gamma$	M
		$x \notin \text{dom}(\Gamma)$	M
red	::=		
		$t_1 \longrightarrow t_2$	t_1 reduces to t_2
fv	::=		
		$x \in \text{FV}(t)$	free variable
aeq	::=		
		$t \equiv_\alpha t'$	alpha equivalence
beq	::=		
		$t \equiv_\beta t'$	beta equivalence

typing ::= $\Gamma \vdash t : T$ Typing rules

judgement ::= red
 fv
 aeq
 beq
 $typing$

user_syntax ::= var
 t
 v
 typ
 ctx
 $terminals$
 $formula$

$t_1 \longrightarrow t_2$ t_1 reduces to t_2

$$\frac{}{(\lambda x. t_{12}) v_2 \longrightarrow [v_2/x] t_{12}} \text{ RED_AX_APP}$$

$$\frac{t_1 \longrightarrow t'_1}{t_1 t \longrightarrow t'_1 t} \text{ RED_CTX_APP_FUN}$$

$$\frac{t_1 \longrightarrow t'_1}{v t_1 \longrightarrow v t'_1} \text{ RED_CTX_APP_ARG}$$

$x \in \text{FV}(t)$ free variable

$$\frac{}{x \in \text{FV}(x)} \text{ FV_VAR}$$

$$\frac{x \in \text{FV}(t_1)}{x \in \text{FV}(t_1 t_2)} \text{ FV_APP_L}$$

$$\frac{x \in \text{FV}(t_2)}{x \in \text{FV}(t_1 t_2)} \text{ FV_APP_R}$$

$$\frac{x \in \text{FV}(t) \quad x \neq y}{x \in \text{FV}(\lambda y. t)} \text{ FV_LAM}$$

$t \equiv_\alpha t'$ alpha equivalence

$$\frac{}{t \equiv_\alpha t} \text{ AEQ_ID}$$

$$\frac{t \equiv_\alpha t'}{t' \equiv_\alpha t} \text{ AEQ_SYM}$$

$$\frac{t \equiv_\alpha t' \quad t' \equiv_\alpha t''}{t \equiv_\alpha t''} \text{ AEQ_TRANS}$$

$$\frac{t_1 \equiv_{\alpha} t'_1 \quad t_2 \equiv_{\alpha} t'_2}{t_1 t_2 \equiv_{\alpha} t'_1 t'_2} \text{AEQ_APP}$$

$$\frac{t \equiv_{\alpha} t'}{\lambda x. t \equiv_{\alpha} \lambda x. t'} \text{AEQ_LAM}$$

$$\frac{x' \notin \text{FV}(t)}{\lambda x. t \equiv_{\alpha} \lambda x'. [x'/x]t} \text{AEQ_SUBST}$$

$t \equiv_{\beta} t'$ beta equivalence

$$\frac{}{t \equiv_{\beta} t} \text{BEQ_ID}$$

$$\frac{t \equiv_{\beta} t'}{t' \equiv_{\beta} t} \text{BEQ_SYM}$$

$$\frac{t \equiv_{\beta} t' \quad t' \equiv_{\beta} t''}{t \equiv_{\beta} t''} \text{BEQ_TRANS}$$

$$\frac{t_1 \equiv_{\beta} t'_1 \quad t_2 \equiv_{\beta} t'_2}{t_1 t_2 \equiv_{\beta} t'_1 t'_2} \text{BEQ_APP}$$

$$\frac{t \equiv_{\beta} t'}{\lambda x. t \equiv_{\beta} \lambda x. t'} \text{BEQ_LAM}$$

$$\frac{}{(\lambda x. t) t' \equiv_{\beta} [\lambda x. t] t'} \text{BEQ_SUBST}$$

$\Gamma \vdash t : T$ Typing rules

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \text{TYPING_VAR}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \lambda x. t : T_1 \rightarrow T_2} \text{TYPING_ABS}$$

$$\frac{\Gamma \vdash t_1 : T_1 \rightarrow T_2 \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash t_1 t_2 : T_2} \text{TYPING_APP}$$

Definition rules: 22 good 0 bad
 Definition rule clauses: 45 good 0 bad