var, x, y term variable

t	$::=$			term
	$\vert$	x		variable
	$\vert$	$\lambda x.t$	bind x in t	lambda
	$\vert$	$t\, t'$		app
	$\vert$	$\textbf{if}\ t\ \textbf{then}\ t'\ \textbf{else}\ t''$		conditional
	$\vert$	true		true
	$\vert$	false		false
	$\vert$	(t)	S	
	$\vert$	$[t/x]t'$	M	
v	$::=$			value
	$\vert$	$\lambda x.t$		lambda
typ, T	$::=$			types
	$\vert$	Bool		bool type
	$\vert$	$T_1 \to T_2$		function types
ctx, Γ	$::=$			typing context
	$\vert$	$\bullet$		empty context
	$\vert$	$\Gamma, x : T$		assumption
$terminals$	$::=$			
	$\vert$	λ		
	$\vert$	$\longrightarrow$		
	$\vert$	$\to$		
	$\vert$	$\in$		
	$\vert$	$\neq$		
	$\vert$	$\equiv_\alpha$		
	$\vert$	$\equiv_\beta$		
	$\vert$	FV		
	$\vert$	$\notin$		
	$\vert$	dom		
	$\vert$	$\vdash$		
	$\vert$	true		
	$\vert$	false		
$formula$	$::=$			
	$\vert$	$judgement$		
	$\vert$	$x \neq x'$	M	
	$\vert$	$x \notin \mathrm{FV}(t)$	M	
	$\vert$	$x : T \in \Gamma$	M	
	$\vert$	$x \notin dom(\Gamma)$	M	
red	$::=$			
	$\vert$	$t_1 \longrightarrow t_2$		t_1 reduces to t_2
fv	$::=$			
	$\vert$	$x \in \mathrm{FV}(t)$		free variable

aeq	::=		
	|	$t \equiv_\alpha t'$	alpha equivalence
beq	::=		
	|	$t \equiv_\beta t'$	beta equivalence
typing	::=		
	|	$\Gamma \vdash t : T$	Typing rules
judgement	::=		
	|	*red*	
	|	*fv*	
	|	*aeq*	
	|	*beq*	
	|	*typing*	
user_syntax	::=		
	|	*var*	
	|	*t*	
	|	*v*	
	|	*typ*	
	|	*ctx*	
	|	*terminals*	
	|	*formula*	

$\boxed{t_1 \longrightarrow t_2}$ t_1 reduces to t_2

$$\frac{}{(\lambda x.t_{12})\ v_2 \longrightarrow [v_2/x]t_{12}} \quad \text{RED\_AX\_APP}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t \longrightarrow t_1'\ t} \quad \text{RED\_CTX\_APP\_FUN}$$

$$\frac{t_1 \longrightarrow t_1'}{v\ t_1 \longrightarrow v\ t_1'} \quad \text{RED\_CTX\_APP\_ARG}$$

$$\frac{}{\textbf{if}\ \mathsf{true}\ \textbf{then}\ t_1\ \textbf{else}\ t_2 \longrightarrow t_1} \quad \text{RED\_IF\_TRUE}$$

$$\frac{}{\textbf{if}\ \mathsf{false}\ \textbf{then}\ t_1\ \textbf{else}\ t_2 \longrightarrow t_2} \quad \text{RED\_IF\_FALSE}$$

$$\frac{t_1 \longrightarrow t_1'}{\textbf{if}\ t_1\ \textbf{then}\ t_2\ \textbf{else}\ t_3 \longrightarrow \textbf{if}\ t_1'\ \textbf{then}\ t_2\ \textbf{else}\ t_3} \quad \text{RED\_IF}$$

$\boxed{x \in \mathrm{FV}\,(t)}$ free variable

$$\frac{}{x \in \mathrm{FV}\,(x)} \quad \text{FV\_VAR}$$

$$\frac{x \in \mathrm{FV}\,(t_1)}{x \in \mathrm{FV}\,(t_1\ t_2)} \quad \text{FV\_APP\_L}$$

$$\frac{x \in \mathrm{FV}\,(t_2)}{x \in \mathrm{FV}\,(t_1\ t_2)} \quad \text{FV\_APP\_R}$$

$$\frac{x \in \mathrm{FV}(t) \quad x \neq y}{x \in \mathrm{FV}(\lambda y.t)} \quad \text{FV\_LAM}$$

$\boxed{t \equiv_\alpha t'}$ alpha equivalence

$$\frac{}{t \equiv_\alpha t} \quad \text{AEQ\_ID}$$

$$\frac{t \equiv_\alpha t'}{t' \equiv_\alpha t} \quad \text{AEQ\_SYM}$$

$$\frac{t \equiv_\alpha t' \quad t' \equiv_\alpha t''}{t \equiv_\alpha t''} \quad \text{AEQ\_TRANS}$$

$$\frac{t_1 \equiv_\alpha t_1' \quad t_2 \equiv_\alpha t_2'}{t_1\ t_2 \equiv_\alpha t_1'\ t_2'} \quad \text{AEQ\_APP}$$

$$\frac{t \equiv_\alpha t'}{\lambda x.t \equiv_\alpha \lambda x.t'} \quad \text{AEQ\_LAM}$$

$$\frac{x' \notin \mathrm{FV}(t)}{\lambda x.t \equiv_\alpha \lambda x'.[x'/x]t} \quad \text{AEQ\_SUBST}$$

$\boxed{t \equiv_\beta t'}$ beta equivalence

$$\frac{}{t \equiv_\beta t} \quad \text{BEQ\_ID}$$

$$\frac{t \equiv_\beta t'}{t' \equiv_\beta t} \quad \text{BEQ\_SYM}$$

$$\frac{t \equiv_\beta t' \quad t' \equiv_\beta t''}{t \equiv_\beta t''} \quad \text{BEQ\_TRANS}$$

$$\frac{t_1 \equiv_\beta t_1' \quad t_2 \equiv_\beta t_2'}{t_1\ t_2 \equiv_\beta t_1'\ t_2'} \quad \text{BEQ\_APP}$$

$$\frac{t \equiv_\beta t'}{\lambda x.t \equiv_\beta \lambda x.t'} \quad \text{BEQ\_LAM}$$

$$\frac{}{(\lambda x.t)\ t' \equiv_\beta [t'/x]t} \quad \text{BEQ\_SUBST}$$

$\boxed{\Gamma \vdash t : T}$ Typing rules

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \quad \text{TYPING\_VAR}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \lambda x.t : T_1 \to T_2} \quad \text{TYPING\_ABS}$$

$$\frac{\Gamma \vdash t_1 : T_1 \to T_2 \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash t_1\ t_2 : T_2} \quad \text{TYPING\_APP}$$

$$\frac{}{\Gamma \vdash \mathsf{true} : \mathsf{Bool}} \quad \text{TYPING\_TRUE}$$

$$\frac{}{\Gamma \vdash \mathsf{false} : \mathsf{Bool}} \quad \text{TYPING\_FALSE}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_1 : \mathsf{Bool} \\ \Gamma \vdash t_2 : T_1 \\ \Gamma \vdash t_3 : T_1\end{array}}{\Gamma \vdash \textbf{if } t_1 \textbf{ then } t_2 \textbf{ else } t_3 : T_1} \quad \text{TYPING\_IF}$$

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Definition rules:        28 good    0 bad
Definition rule clauses: 55 good    0 bad
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