var, x, y	term variable			
t	::=			term
	|	x		variable
	|	$\lambda x.t$	bind x in t	lambda
	|	$t\, t'$		app
	|	$\mathbf{z}$		zero
	|	$\mathbf{s}\,(t)$		successor
	|	$\mathbf{rec}\,(t, t_0, x.y.t_1)$		recursion
	|	(t)	S	
	|	$[t/x]t'$	M	
v	::=			value
	|	$\lambda x.t$		lambda
typ, T	::=			types
	|	Nat		natural numbers
	|	$T_1 \to T_2$		function types
ctx, Γ	::=			typing context
	|	$\bullet$		empty context
	|	$\Gamma, x : T$		assumption
terminals	::=			
	|	λ		
	|	$\longrightarrow$		
	|	$\to$		
	|	$\in$		
	|	$\neq$		
	|	$\equiv_\alpha$		
	|	$\equiv_\beta$		
	|	FV		
	|	$\notin$		
	|	*dom*		
	|	$\vdash$		
	|	true		
	|	false		
formula	::=			
	|	*judgement*		
	|	$x \neq x'$	M	
	|	$x \notin \mathrm{FV}\,(t)$	M	
	|	$x : T \in \Gamma$	M	
	|	$x \notin \mathit{dom}\,(\Gamma)$	M	
red	::=			
	|	$t_1 \longrightarrow t_2$		t_1 reduces to t_2
fv	::=			
	|	$x \in \mathrm{FV}\,(t)$		free variable

aeq	::=		
	|	$t \equiv_\alpha t'$	alpha equivalence
beq	::=		
	|	$t \equiv_\beta t'$	beta equivalence
typing	::=		
	|	$\Gamma \vdash t : T$	Typing rules
judgement	::=		
	|	*red*	
	|	*fv*	
	|	*aeq*	
	|	*beq*	
	|	*typing*	
user_syntax	::=		
	|	*var*	
	|	*t*	
	|	*v*	
	|	*typ*	
	|	*ctx*	
	|	*terminals*	
	|	*formula*	

$\boxed{t_1 \longrightarrow t_2}$ t_1 reduces to t_2

$$\frac{}{(\lambda x.t_{12})\ v_2 \longrightarrow [v_2/x]t_{12}} \quad \text{RED\_AX\_APP}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t \longrightarrow t_1'\ t} \quad \text{RED\_CTX\_APP\_FUN}$$

$$\frac{t_1 \longrightarrow t_1'}{v\ t_1 \longrightarrow v\ t_1'} \quad \text{RED\_CTX\_APP\_ARG}$$

$\boxed{x \in \text{FV}\,(t)}$ free variable

$$\frac{}{x \in \text{FV}\,(x)} \quad \text{FV\_VAR}$$

$$\frac{x \in \text{FV}\,(t_1)}{x \in \text{FV}\,(t_1\ t_2)} \quad \text{FV\_APP\_L}$$

$$\frac{x \in \text{FV}\,(t_2)}{x \in \text{FV}\,(t_1\ t_2)} \quad \text{FV\_APP\_R}$$

$$\frac{\begin{array}{c} x \in \text{FV}\,(t) \\ x \neq y \end{array}}{x \in \text{FV}\,(\lambda y.t)} \quad \text{FV\_LAM}$$

$\boxed{t \equiv_\alpha t'}$ alpha equivalence

$$\frac{}{t \equiv_\alpha t} \quad \text{AEQ\_ID}$$

$$\frac{t \equiv_\alpha t'}{t' \equiv_\alpha t} \quad \text{AEQ\_SYM}$$

$$\frac{t \equiv_\alpha t' \quad t' \equiv_\alpha t''}{t \equiv_\alpha t''} \quad \text{AEQ\_TRANS}$$

$$\frac{t_1 \equiv_\alpha t_1' \quad t_2 \equiv_\alpha t_2'}{t_1\ t_2 \equiv_\alpha t_1'\ t_2'} \quad \text{AEQ\_APP}$$

$$\frac{t \equiv_\alpha t'}{\lambda x.t \equiv_\alpha \lambda x.t'} \quad \text{AEQ\_LAM}$$

$$\frac{x' \notin \mathrm{FV}\,(t)}{\lambda x.t \equiv_\alpha \lambda x'.[x'/x]t} \quad \text{AEQ\_SUBST}$$

$\boxed{t \equiv_\beta t'}$ beta equivalence

$$\frac{}{t \equiv_\beta t} \quad \text{BEQ\_ID}$$

$$\frac{t \equiv_\beta t'}{t' \equiv_\beta t} \quad \text{BEQ\_SYM}$$

$$\frac{t \equiv_\beta t' \quad t' \equiv_\beta t''}{t \equiv_\beta t''} \quad \text{BEQ\_TRANS}$$

$$\frac{t_1 \equiv_\beta t_1' \quad t_2 \equiv_\beta t_2'}{t_1\ t_2 \equiv_\beta t_1'\ t_2'} \quad \text{BEQ\_APP}$$

$$\frac{t \equiv_\beta t'}{\lambda x.t \equiv_\beta \lambda x.t'} \quad \text{BEQ\_LAM}$$

$$\frac{}{(\lambda x.t)\ t' \equiv_\beta [t'/x]t} \quad \text{BEQ\_SUBST}$$

$\boxed{\Gamma \vdash t : T}$ Typing rules

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \quad \text{TYPING\_VAR}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \lambda x.t : T_1 \rightarrow T_2} \quad \text{TYPING\_ABS}$$

$$\frac{\Gamma \vdash t_1 : T_1 \rightarrow T_2 \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash t_1\ t_2 : T_2} \quad \text{TYPING\_APP}$$

$$\frac{}{\Gamma \vdash \mathbf{z} : \mathsf{Nat}} \quad \text{TYPING\_Z}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat}}{\Gamma \vdash \mathbf{s}\,(t) : \mathsf{Nat}} \quad \text{TYPING\_S}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat} \quad \Gamma \vdash t_0 : T \quad \Gamma \vdash x : \mathsf{Nat} \quad \Gamma \vdash y : T \quad \Gamma \vdash t_1 : T}{\Gamma \vdash \mathbf{rec}\,(t, t_0, x.y.t_1) : T} \quad \text{TYPING\_REC}$$

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Definition rules:         25 good    0 bad
Definition rule clauses: 54 good    0 bad
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