DD2552 Seminar 4: Data types

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Course material

- PFPL chapters 10 and 11, finite data types
- PFPL chapter 9, natural numbers
- PFPL chapter 15, (co)inductive data types

Why data types?

- convenient in many applications - otherwise lots of encoding
- progress/preservation holds once and for all
- domain modeling tool

Products

- mathematical “tuples” at programming language level
- basic form of structured data
- ubiquitous

Binary product syntax

$$
\begin{array}{lll}
T & ::= & \\
 & | & \mathsf{unit} \\
 & | & \mathsf{prod}\,(T_1, T_2) \\
 & & \\
e & ::= & \\
 & | & \mathbf{triv} \\
 & | & \mathsf{pair}\,(e_1, e_2) \\
 & | & \mathsf{pr}\,[\,\mathbf{l}\,](e) \\
 & | & \mathsf{pr}\,[\,\mathbf{r}\,](e)
\end{array}
$$

General product syntax

$$
\begin{array}{lll}
T & ::= & \\
 & | & i_1 \rightsquigarrow T_1 * \ldots * i_n \rightsquigarrow T_n \\
 & & \\
e & ::= & \\
 & | & < i_1 \rightsquigarrow e_1, \ldots, i_n \rightsquigarrow e_n > \\
 & | & \mathsf{pr}\,[i](e)
\end{array}
$$

Sum types

- representation of finite choices
- choice determines data structure
- leaf vs. branch, something vs. nothing

Natural numbers and recursors

t	$::=$			term
	$\mid$	x		variable
	$\mid$	$\lambda x.t$	bind x in t	lambda
	$\mid$	$t\,t'$		app
	$\mid$	$\mathbf{z}$		zero
	$\mid$	$\mathbf{s}\,(t)$		successor
	$\mid$	$\mathbf{rec}\,(t, t_0, x.y.t_1)$		recursion
	$\mid$	(t)	S	
	$\mid$	$[t/x]t'$	M	
$typ,\ T$	$::=$			types
	$\mid$	Nat		natural numbers
	$\mid$	$T_1 \to T_2$		function types

Typing numbers and recursors

$$\frac{}{\Gamma \vdash \mathbf{z} : \mathsf{Nat}} \ \textsc{typing\_z}$$

$$\frac{\Gamma \vdash t : \mathsf{Nat}}{\Gamma \vdash \mathbf{s}\,(t) : \mathsf{Nat}} \ \textsc{typing\_s}$$

$$\frac{\begin{array}{l}\Gamma \vdash t : \mathsf{Nat} \\ \Gamma \vdash t_0 : T \\ \Gamma \vdash x : \mathsf{Nat} \\ \Gamma \vdash t_1 : T\end{array}}{\Gamma \vdash \mathbf{rec}\,(t, t_0, x.y.t_1) : T} \ \textsc{typing\_rec}$$