Karl Palmskog (https://setoid.com) is the document author.

ITPPV Homework 6

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1 Informal Propositional Logic Proof System

Study the propositional logic proof system defined informally at `https://kth-step.github.io/itppv-course/lectures/propositional.pdf`. A proof in the system might be written as follows:

```
p -> (p -> q) , p |- q
1 p -> (p -> q) premise
2 p premise
3 p -> q impe 2, 1
4 q impe 2, 3
```

Here is another proof using boxes:

```
q |- p -> q
[
 1 p assumption
 2 q premise
]
3 p -> q impi 1-2
```

2 Formal Propositional Logic Proof System in HOL4

Download the files `proofScript.sml` and `proofMetaScript.sml` from `https://github.com/kth-step/itppv-course/tree/master/homeworks/hw6-supplementary`. `proofScript.sml` defines the deep embedding of propositional logic and some proof rules. `proofMetaScript.sml` proves that the defined rules are sound. Use `Holmake` to build theories for both files. You will notice that `proofMetaScript.sml` is cheating (not on soundness, but in some theorem outlines).

3 Soundness of All Standard Rules

Extend the HOL4 proof rule relation in `proofScript.sml` to encompass all rules from the informal list. Fill in tactics instead of `cheat` in `proofMetaScript.sml`, and use the corresponding theorems to obtain the full soundness proof.

4 Extended Propositional Logic Syntax and Soundness

Extend the definition of propositional logic syntax in `proofScript.sml` to include the "Sheffer's stroke" operator as defined in `https://www.st-andrews.ac.uk/~slr/sheffer.pdf`. Extend the semantic mapping of propositions (`prop_of` in `proofMetaScript.sml`) to capture the meaning of the operator. Finally, define meaningful introduction and elimination rules for the operator (the paper gives some different ideas on how to do this), and prove in HOL4 that the whole system is sound.

5 Optional: Encoding a Proof

As an optional task, encode a simple propositional statement and its proof (i.e., a "claim"), such as one of those above. Prove that the claim is well-formed according to your proof system relation. Use your proof system soundness proof to derive a corresponding proof about HOL4 `bool`s.